AESB2320 Part 1 Exam 12 March 2019

1. a) According to BSLK Fig 6.4-2, flow starts to deviate from Darcy's law ("Blake-Kozeny eq.") at $\left(\frac{D_p G_0}{\mu}\right)\left(\frac{1}{1-\varepsilon}\right) \sim 10$, and is fully turbulent at 1000. The text says just below Eq. 6.4-7 (BSLK), that it applies for $\left(\frac{D_p G_0}{\mu}\right)\left(\frac{1}{1-\varepsilon}\right) > 1000$.

$G_0 = \rho V_0 = \rho Q / A$,

$$\frac{D_p G_0}{\mu} \frac{1}{1-\varepsilon} = \frac{(10^{-4})1000\,Q}{0.005(\pi(0.005)^2} \frac{1}{1-0.35} = 15671\,Q = 1000$$

$\underline{Q = 0.00255 \text{ m}^3/\text{s}}$ (about 64 liters/sec!)

$$V_0 = \frac{0.00255}{\pi(0.005)^2} = 32.5 \text{ m/s} \quad (!)$$

b) since we know V_0, we can solve either eq. 6.4-11 or 12 (BSLK) for $(\mathcal{P}_0 - \mathcal{P}_L)$. Using 6.4-12,

$$\frac{\mathcal{P}_0 - \mathcal{P}_L}{0.15} = 150 \frac{0.005 \cdot 32.5}{(0.0001)^2} \frac{(0.65)^2}{(0.35)^3} + \frac{7}{4} \frac{1000(32.5)^2}{0.0001} \frac{0.65}{(0.35)^3}$$

$$= 2.40 \cdot 10^{10} + 2.80 \cdot 10^{11} = 3.04 \cdot 10^{11}$$

Not incl. in Eq. 6.4-11, which is also OK.

$(\mathcal{P}_0 - \mathcal{P}_L) = 4.56 \cdot 10^{10}$ Pa/m $\leftarrow 4.20 \cdot 10^{10}$ using Eq. 6.4-11

If one chose $\left(\frac{D_p G_0}{\mu}\right)\left(\frac{1}{1-\varepsilon}\right) = 10$ instead, (which isn't fully turbulent), $Q = 0.00638$ m³/s, $V_0 = 3.25$ m/s,

$$\frac{\mathcal{P}_0 - \mathcal{P}_L}{0.15} = 2.40 \cdot 10^9 + 2.80 \cdot 10^9 = 5.20 \cdot 10^9 \text{ Pa/s}$$

(Many students used the condition for turbulence in an empty tube of 1 cm dia. This is a packed bed, which only happens to be packed in a cylindrical tube.)

same terms in balance, same B.C. on τ at $x=0$

2. The momentum balance here is the same for the falling film of Newtonian fluid. τ_{xz} is at a maximum at the wall, where $\tau_{xz} = \rho g \cos\beta\, \delta$. If this equals τ_0, the peanut butter starts to flow. BSL1 Eq. 2.2-11

$(1100)(9.8?)(0.707)\delta = 27 \rightarrow \delta = 3.52$ mm

Actually, other peanut butters have a larger yield stress. Still, the artist would be wise to leave his surface horizontal.

(Solution to problem 3 on next p.)

4. b) We can answer this first. This problem has a sudden contraction (going into the slit), and a kinetic-energy change not accounted for in the equations for turbulence in ch. 6. see next p.

a) Put surface "1" just outside the slit in the ocean, and surface 2 just inside the outlet. Use Eq. 7.5-11 or 12.

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}v^2$ (v = velocity in slit)

$g(h_2 - h_1) = 0$

$P_1 = 1\text{ atm} + \rho g(3.05) = 1\text{ atm} + 1037(9.82)(3.05) = 31059\text{ Pa} + 1\text{ atm}$

$P_2 = 1\text{ atm}$; $(P_2 - P_1)/\rho = 31059/1037 = 29.95$

$\hat{W}_m = 0$ What is Re? Using hydraulic-radius approx for slit, $D_h = 4B$. Since $Q = v\,2B\,W$, $v = \frac{Q}{2BW}$

$Re = \frac{D_h v \rho}{\mu} = \frac{4B(0.189)(1037)}{0.001(2B)(60.96)} = 6430$, indep of $2B$

therefore $f \approx 0.0098$ (BSLK Fig 6.2-2)

$\frac{1}{2}v^2 \frac{L\cdot 4}{D_h} f = 2v^2 \frac{(0.0508)}{4B}(0.0078) = 0.000996\frac{v^2}{4B} = \frac{0.000996 v^2}{0.0062/v} = 0.161\,v^3$

But since $v = \frac{Q}{2BW}$, $2B = \frac{Q}{W}\frac{1}{v} = \frac{0.189}{60.96}\frac{1}{v} = 0.0031/v$, $4B = \underline{0.0062/v}$

sudden contraction: $e_v = 0.45 \leftarrow$ (Table 7.5-1)

$$\left\{\begin{array}{l} \frac{1}{2}v^2 - 29.95 = -0.161\,v^3 - \frac{1}{2}v^2(0.45) \quad \boxed{\text{I}} \\ \text{with } 2B = 0.0031/v \end{array}\right.$$

Since both v^3 and v^2 are in this equation, our simple iterative method won't work. Using "solver" in Excel, I find $v = 4.53$ m/s, $2B = 0.000685$ (685 μm).

see note next p.

3. With two fluid layers, we need 4 BC

1) $\tau_{rz}^{I} = 0$ at $r = R_2$ (vapor-liquid interface at $r = R_2$)

2) continuity of τ_{rz} at $r = R_1$, i.e., $\tau_{rz}^{I} = \tau_{rz}^{II}$ if call layers I + II

3) " v_z " i.e., $v_z^{I} = v_z^{II}$

4) $v_z^{II} = 0$ at $r = R$ (no slip at $r = R$)

Note on problem 4:

The eq. we learn in ch 6 (when learning about friction factors includes Δp and hydrostatics (in $\Delta \mathcal{P}$) and drag on the walls (through the friction factor).

What's new here is the constriction at the entrance and kinetic energy. How important are those terms here? Plug final answer for v into Eq. I fr. prev. page

$$\frac{1}{2}(4.53)^2 - 29.95 = -0.161(4.53)^3 - \frac{1}{2}(4.53)^2(0.45)$$

$$\underbrace{10.26}_{K.E.} - \underbrace{29.95}_{\Delta p} = \underbrace{-14.96}_{\text{drag on wall}} - \underbrace{4.61}_{\text{constriction}}$$

About half the energy goes into the new terms. As a result, the slit is 685 μm instead of 520 μm accounting for turbulence (P.S.7) or 394 μm assuming laminar flow (P.S. 6).